

3 points problems

1. A triangle has integer side-lengths. One side is 9 and another is 1.

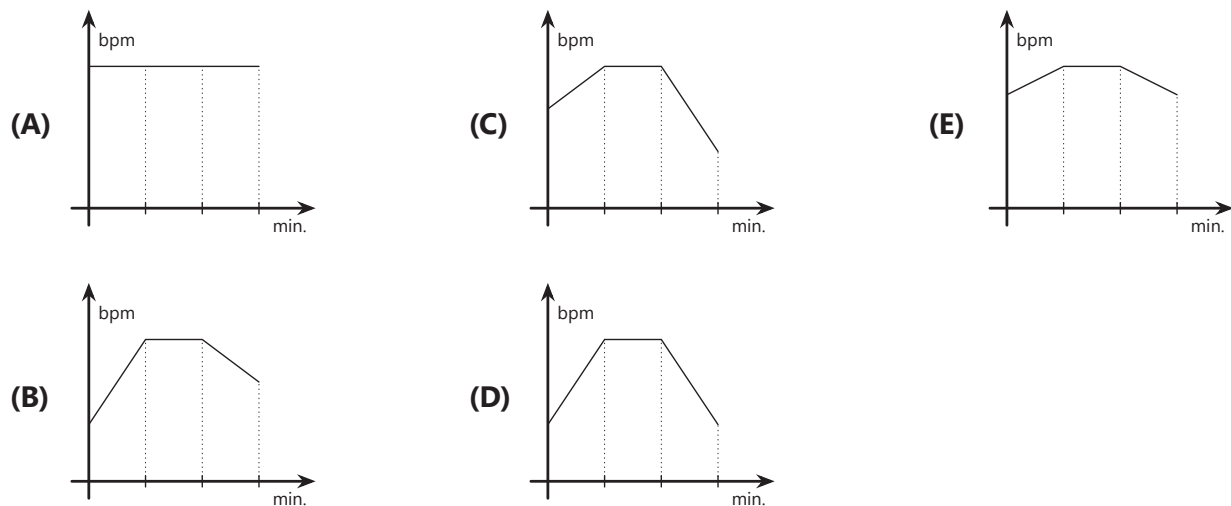
What is the length of the third side?

- (A) 5 (B) 7 (C) 9 (D) 11 (E) 13

2. During a 30-minute jog, Mia's smartwatch gave this report:

- For the first ten minutes, her heart rate increased by 4 beats per minute (bpm) every minute.
- For the next ten minutes, her heart rate stayed constant.
- For the last ten minutes, her heart rate decreased by 2 bpm every minute.

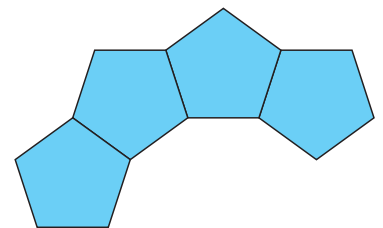
Which of the following shows the graph of her heart rate?



3. Tiles in the shape of regular pentagons are arranged side by side, sharing edges, to form a ring. The figure shows four of these tiles.

How many tiles are there in the complete ring?

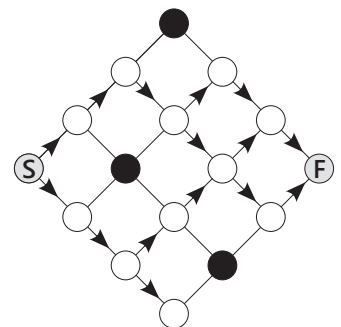
- (A) 10 (D) 14
(B) 11 (E) 15
(C) 12



4. Indiana wants to walk from S to F. He can only walk along the paths marked and only in the directions indicated by the arrows. He must also avoid the black stones.

How many different paths can he take?

- (A) 5 (D) 8
(B) 6 (E) 9
(C) 7



5. What is the largest number you can get by replacing the four blanks in the expression with the four digits 2, 0, 2 and 6? $(\square + \square)(\square - \square)$

(A) 2^4 (B) 2^6 (C) 2^8 (D) 2^{10} (E) 2^{12}

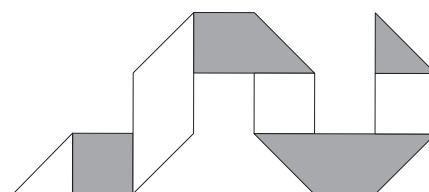
6. A store has the following special offer: If you buy three items, you will get the cheapest item for free. Julia selects six pairs of socks, each at a different price. Individually they are priced at 2.90, 3.10, 3.50, 4.30, 4.60 and 4.90 euros.

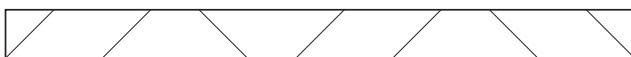
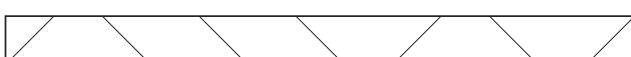
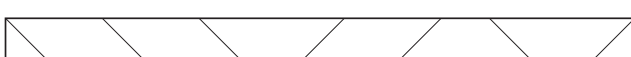
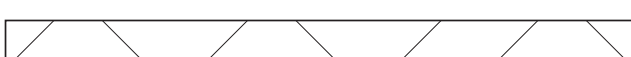
What is the maximum total value of the two pairs of socks she can get for free?

(A) 6.60 euros (B) 7.20 euros (C) 7.40 euros (D) 7.70 euros (E) 8.10 euros

7. Ali made seven folds in a strip of paper with a white side and a dark side, as seen in the figure. He then unfolded the paper.

What does the white side of the paper look like after unfolding?

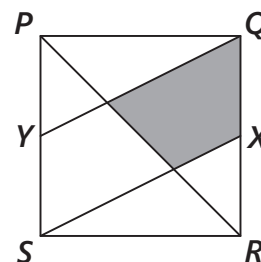


- (A) 
- (B) 
- (C) 
- (D) 
- (E) 

8. The diagram shows a square $PQRS$. The points X and Y are the midpoints of the sides QR and PS , respectively.

What fraction of the square is shaded?

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$



9. A hotel has nine vacant rooms. Each room can accommodate three or four people. A group of 30 people is going to stay at the hotel and will completely fill all the vacant rooms.

How many rooms have a capacity of four people?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

10. How many three-digit numbers, abc , are there such that $a = \left(\frac{b}{c}\right)^2$?

(A) 4 (B) 8 (C) 9 (D) 10 (E) 16

4 points problems

11. The number $\underbrace{333\dots3}_{2026 \text{ digits}}$ is divided by 33.

What is the sum of the digits of the result?

- (A) 1111 (B) 1013 (C) 2025 (D) 2026 (E) 3039

12. Two points P and Q are randomly placed on a line segment AB , with neither at the mid-point.



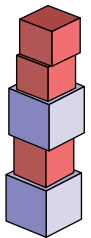
What is the probability that the line segment PQ contains the mid-point of AB ?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

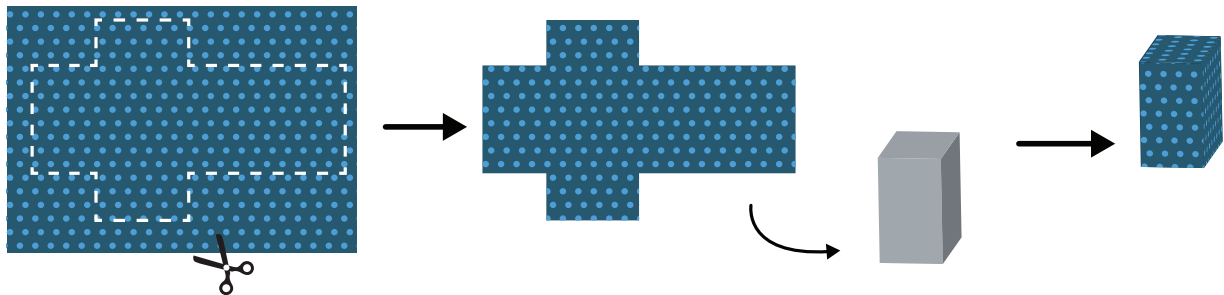
13. We wish to build a tower out of two kinds of cubic blocks. One kind is 5 cm tall and the other is 4 cm tall. We have as many of each kind of block as may be required.

What is the largest integer number of centimetres that **CANNOT** be the height of a tower that can be built with the blocks?

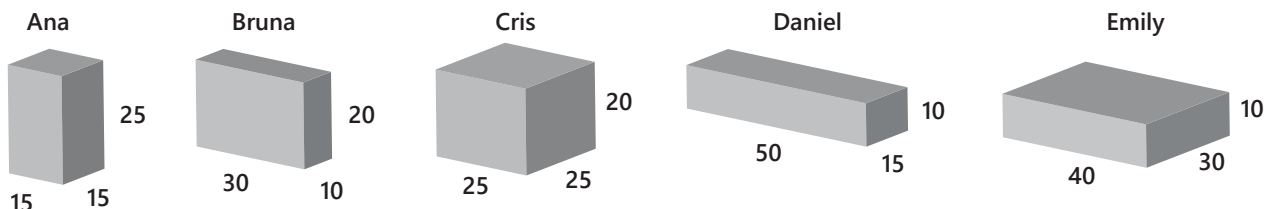
- (A) 7 cm (B) 11 cm (C) 17 cm (D) 37 cm (E) 101 cm



14. Vitória wraps gifts in her store using wrapping paper sheets that measure 80 cm x 60 cm. To wrap a gift box, she cuts a single piece of the wrapping paper and attaches it to the sides of the box, as shown.



The edge lengths, in centimetres, of five friends' gift boxes are shown below.



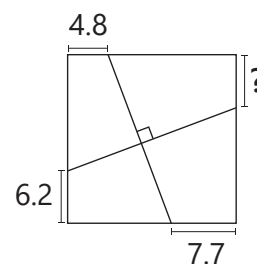
Who can **NOT** have his/her gift box completely wrapped?

- (A) Ana (B) Bruna (C) Cris (D) Daniel (E) Emily

15. The picture shows a square and two lines that are perpendicular. The lengths of three line segments are given.

What is the length of the line segment with the question mark?

- (A) 5.6 (B) 5.9 (C) 6.1 (D) 6.3 (E) 6.6



16. Two standard dice are rolled and the product of the numbers obtained is recorded.

- Anne gets a point if the product is divisible by 4.
- Drew gets a point if the product is divisible by 6.

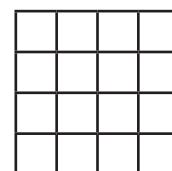
What is the probability that both Anne and Drew get a point?

- (A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{5}{36}$ (D) $\frac{7}{36}$ (E) $\frac{2}{9}$

17. Amir has a 4×4 grid made up of 16 squares. He wants to use a cutter to make straight cuts on this grid so that no square remains intact.

What is the smallest number of cuts he must make?

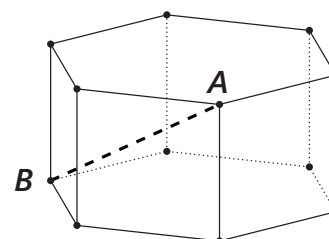
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6



18. This hexagonal prism has two regular hexagons and six squares as faces. All edges are 1 unit long.

What is the length of the line segment AB shown?

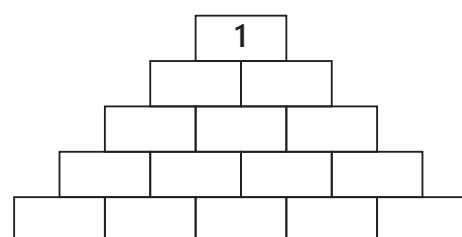
- (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{5}$ (E) $\sqrt{6}$



19. Ali wants to fill the pyramid shown from bottom to top with the numbers -1 and 1 so that each number, apart from those in the bottom row, is equal to the product of the two numbers directly below it. In the end, the number at the top of the pyramid must be 1 .

In how many ways can he do this?

- (A) 8 (B) 16 (C) 18 (D) 20 (E) 32



20. The real numbers a and b are such that $9^a = 11^b = 9801$.

What is $\frac{1}{a} + \frac{1}{b}$?

- (A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) 1 (D) 2 (E) 3

5 points problems

21. The integers $1, 2, \dots, 40$ are written on a blackboard. David performs 39 operations on these numbers. On the k -th operation:

- if k is not a multiple of 7, he picks any two numbers a, b , erases them, and writes $a + b - 1$.
- if k is a multiple of 7, he picks any two numbers a, b , erases them, and writes $a + b + 5$.

What number remains at the end?

- (A) 781 (B) 801 (C) 811 (D) 819 (E) 821

22. The sum of 15 consecutive natural numbers is the same as the sum of the next 9 natural numbers. What is the smallest of these 24 numbers?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

23. Oleg threw 100 standard dice and multiplied all the numbers that appeared on the upper faces. The resulting product was 6^{70} .

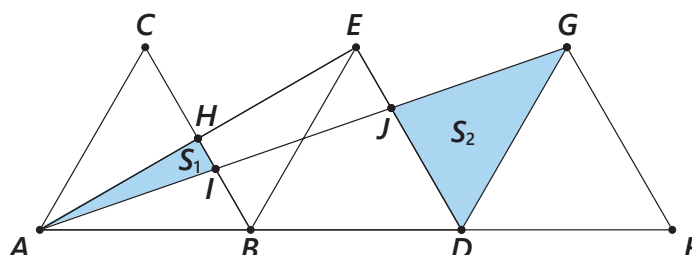
What is the smallest number of times the number 6 could have appeared?

- (A) 10 (B) 12 (C) 24 (D) 30 (E) 35

24. Three congruent equilateral triangles are drawn on the line segment AF , as shown in the diagram. Denote the area of $\triangle AHI$ as S_1 and the area of $\triangle DGJ$ as S_2 .

What is the ratio $S_1 : S_2$?

- (A) 1 : 3 (D) 2 : 3
(B) 1 : 4 (E) 3 : 5
(C) 1 : 5



25. A function f has the property that for every real number x ,

$$f(x + 10) = f(x) \quad \text{and} \quad f(6 - x) = -f(x)$$

The value of $f(27) = 9$.

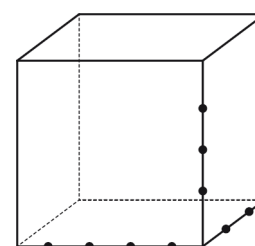
What is the value of $f(9) + f(13)$?

- (A) - 27 (B) - 9 (C) - 3 (D) 3 (E) 9

26. Nine points have been selected on the edges of a cube, as shown in the diagram.

How many triangular pyramids are there whose vertices are among these nine points?

- (A) 24 (C) 48 (E) 72
(B) 36 (D) 60

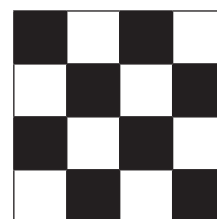


27. For a natural number n , let a_n denote the largest integer less than or equal to $\sqrt{n}$.

What is the value of $a_1 - a_2 + a_3 - a_4 + a_5 - a_6 + \dots + a_{2025} - a_{2026}$?

- (A) 0 (D) 22
(B) 2026 (E) -22
(C) -2026

28. On a 4×4 board, coloured as shown, we want to make all the squares white by repeatedly performing the following operation: choose any 2×2 square and switch the colour of these 4 squares.



What is the minimum number of times this operation must be performed?

- (A) 4 (D) 16
(B) 6 (E) It is not possible to do this.
(C) 8

29. For a number $x > 0$, define $\sqrt[n]{x}$, the *triangular root* of x , to be the value $s > 0$ such that $\frac{s(s+1)}{2} = x$.

Which of the following is always equal to $\sqrt[n]{4x} - \sqrt[n]{x}$?

- (A) $2\sqrt[n]{x}$ (D) $\sqrt[n]{x^2 + x}$
(B) $4\sqrt[n]{x} - 1$ (E) $\sqrt[n]{x^2}$
(C) $3\sqrt[n]{x}$

30. There are $2n$ doors in front of us. Each door can be unlocked with only one key. Each of the n keys we have unlocks two doors, but we don't know which ones.

What is the maximum number of attempts needed to determine which key opens which door?

- (A) $2n - 2$ (D) $n^2 - 1$
(B) $2n^2$ (E) $n^2 + n - 2$
(C) $n^2 + n$